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Irrotationality of an Incompressible Fluid in Stationary Waves

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Abstract

In the present paper, I have investigated Irrotationality of an Incompressible fluid in stationary waves. We have investigated path line, stream line, velocity potential, complex potential, phase velocity.

Keywords: Stream function, Complex potential.

Nomenclature: η = Simple harmonic progressive wave, u = Velocity along x axis, v = Velocity along y axis , ϕ = Velocity potential , ψ = Stream function , W = Complex potential , y = vertical axis coordinate , x = Axial coordinate.

Introduction

In the present paper, I have investigated Irrotationality of an Incompressible fluid in stationary waves. Attempt have been made by several researches, i.e Banerjee Mihir B. & Shandil R. G. [1] Investigated conjecture in heterogeneous shear flow instability of modified S waves. Aparicio N. D. and Atkinson C [2] investigate plain dynamic crack propagation in a non – homogeneous visco- elastic strip. Bhaumick, Rana and Dass Bikas [3] investigated steady state thermal stresses in an infinite elastic medium containing an annular crack. Bois P. A [4] investigated Boussineso wave theory in fluid mixture with application to the cloudy atmosphere. Kumar Rajneesh and Singh Baljeet [5] investigated wave propagation in a generalized thermo elastic body with streach. Kyani V. K. [6] investigated the dispersion of long waves in an infinite stressed multilayered crust. Nicolaou D. and Stevenson T. N. [7] investigated internal waves around a disturbance in a fluid with arbitrary stratification and background shear flow. Rajhans B. K. and Samal S. K. [9] investigated the diffraction of compressible waves by a fluid cylinder in a homogeneous medium. We have investigated path line, stream line, velocity potential, complex potential motion and phase velocity.

Formulation of the Problem

Suppose the waves which remain stationary the surface moves vertically only at the surface of a canal of uniform depth h with parallel vertical walls at right angles to the ridges and hollows.

The fluid is incompressible and the motion produced by natural forces is irrotational the velocity potential ϕ exists .

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \qquad \text{.........(1)}$$

with boundary condition

$$\phi\,(x, 0) = x\,\sigma\rho\,\vartheta + \eta \qquad \text{.........(2)}$$

$$\phi\,(x, \pi) = 0 \qquad \text{........(3)}$$

$$\frac{\partial \Phi}{\partial x} = 0 \quad \text{at } y=0,\ x=0 \qquad \text{.........(4)}$$

$$\frac{\partial \Phi}{\partial x} = 0 \quad \text{at } y=0,\ x=\pi \qquad \text{.........(5)}$$

Also the equation for stationary wave is given by

$$\eta = a \sin nx \cos mt \qquad \text{..........(6)}$$

Solution of the Problem

Let the solution of equation (1) be taken as

$$\phi(x, y) = X(x)\ Y(y) \qquad \text{............(7)}$$

the corresponding differential equation is

$$\frac{d^2 x}{dx^2} + \xi^2 X = 0 \qquad \text{...........(8)}$$

$$\frac{d^2 x}{dx^2} + \xi^2 Y = 0 \qquad \text{............(9)}$$

$$X = c_1 \cos \xi x + c_2 \sin \xi x \qquad \text{............(10)}$$

$$Y = c_3 \cosh \xi y + c_4 \sinh \xi y \qquad \text{............(11)}$$

The solution of (8) and (9) is

$$\phi(x,y) = (c_1 \cos \xi x + c_2 \sin \xi x)\,(c_3 \cosh \xi y + c_4 \sinh \xi y) \qquad \text{.............(12)}$$

$\phi(x,\pi) = (c_1 \cos \xi x + c_2 \sin \xi x)(c_3 \cosh \xi\pi + c_4 \sinh \xi\pi)$

$$\phi(x,y) = (\ddot{\Theta}_1 \cos \xi x + \ddot{\Theta}_2 \sin \xi x) \frac{\sinh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{............(13)}$$

$$\left(\frac{\partial \Phi}{\partial x}\right) = \xi(-\ddot{\Theta}_1 \sin \xi x + \ddot{\Theta}_2 \cos \xi x) \frac{\sinh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{............(14)}$$

$$\left(\frac{\partial \Phi}{\partial x}\right)_{(0,\,y)} = \xi\, \ddot{\Theta}_2 \frac{\sinh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{............(15)}$$

$$\phi(x,y) = \sum_{n=1}^{\infty} (\text{Ч}_n) \cos nx \frac{\sinh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{.............(16)}$$

$$\text{Ч}_n = \sum_{n=1}^{\infty} \left[\frac{2\sigma\rho\vartheta}{n^2\pi} \{(-1)^n - 1\}\right]$$

$$\Phi(x,y) = \sum_{n=1}^{\infty} \left[\frac{2\sigma\rho\vartheta}{n^2\pi} \{(-1)^n - 1\}\right] \cos nx \frac{\sinh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{.............(17)}$$

$$U = \sum_{n=1}^{\infty} \left[\frac{2\sigma\rho\vartheta}{n^2\pi} \{(-1)^n - 1\}\right] \sin nx \frac{\sinh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{..............(18)}$$

$$V = \sum_{n=1}^{\infty} \left[\frac{2\sigma\rho\vartheta}{n^2\pi} \{(-1)^n - 1\}\right] \sin nx \frac{\cosh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{..........(19)}$$

$$\Psi = -\sum_{n=1}^{\infty} \left[\frac{2\sigma\rho\vartheta}{n^2\pi} \{(-1)^n - 1\}\right] \sin nx \frac{\cosh \xi(\pi - y)}{\sinh \xi\pi} \qquad \text{..............(20)}$$

$$W = \sum_{n=1}^{\infty} \left[\frac{2\sigma\rho\vartheta}{n^2\pi} \{(-1)^n - 1\}\right] [\cos nx \sinh n(\pi - y) - i \sin nx \cosh n(\pi - y)] \qquad \text{................(21)}$$

The stream line is

$$\text{Sinx} = \cosh n(\pi - y)\,.\Upsilon \qquad \text{................(22)}$$

And $z = A_1$ (constant)

$$\text{Also curl } \hat{q} = \sum_{n=1}^{\infty} \left[\frac{2\sigma\rho\vartheta}{n^2\pi} \{(-1)^n - 1\}\right] [(-n \sin nx) \cosh n(\pi - y) + n \sin nx \cosh n(\pi - y)]$$

$$= 0$$

The motion is irrotational.

$$\text{Also path} = \frac{P}{Q} = \cot nx \coth n(\pi - y) \qquad \text{.............(23)}$$

Result and Discussion

In the present paper , we have investigated velocity potential, velocity components, stream function, complex potential and stream line of motion, path of particle of a liquid of an Incompressible fluid in stationary waves given by the equation (17) , (18), (19), (20), (21), (22), (23) , .

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